Ultra Wideband Channel Estimation based on Kalman Filter Compressed Sensing

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Abstract— in this paper, a novel time-varying channel estimation approach based on Kalman filter compressive sensing is proposed for the high sampling problem of ultra wideband (UWB) system considering the sparse of the channel impulse response. The direct sequence UWB signal is formulated to the mathematical model of compressed sensing after down sampling. The receiver recovery the channel impulse response by Kalman filter compressive sensing algorithm. The simulation results demonstrate that the proposed scheme can reduce the required sampling points and improve the accuracy of estimation algorithm.

Keywords- ultra wideband; channel estimation; compressed sensing; kalman filter

I. INTRODUCTION

Ultra-wideband (UWB) communication [1], [2] is a fast emerging technology since the Federal Communication Commission released a spectral mask in the spring of 2002. The major reason for UWB technology to receive much attention is its promising ability to provide low-power consumption, high bit rate and multipath resolution, and coexist with the narrow-band system by trading bandwidth for a reduced transmits power.

In the impulse radio UWB (IR-UWB) systems, the duration of pulse is ultra-short, typically on the order of nanoseconds. On one hand, the ultra-short impulses make it possible to resolve and combine signal echoes with path length differential down to 1 ft exploiting the diversity inherent in the multipath channel and improving the position accuracy. On the other hand, the new technical [3] challenges are posed: (1) analog-to-digital converters (ADCs) working at the Nyquist rate are in general very expansive and power demanding; (2) the synchronization which is accomplished at the scale of sub nanosecond duration is extremely complex; (3) capture a sufficient amount of the rich multipath diversity need accuracy channel estimation. Compare to the transmitter easily implement, the IR-UWB receivers are too complex.1

Traditionally, the IR-UWB channel estimation [4] includes two approaches. One is a date-aided framework which employs analog delay units to yield a symbol-long estimate of compose pulse-multipath channel. The penalty of the analog delay unit is the high power consume. Another is maximum likelihood (ML) estimation of the optimal value of the path gains and path delays, which typically need a tens GHz order sampling rate. The most important problem of all the algorithms mentioned above is that they all modeled the channel parameters as quasi-static. In other word, the channel impulse response was assumed to be time-invariant which was unrealistic in the real wireless environment.

The emerging theory of compressed sensing (CS) [5] provides new approaches for practical IR-UWB receiver design. When the short duration pulses in the IR-UWB system propagate through the multipath channels, the received signals remain sparse in time domain. The sampling rate can be reduced to sub-Nyquisit rate and the receiver can reconstruct the initial signal with high probability.

In this paper, we propose an IR-UWB channel estimation approach which employs Kalman filter compressive sensing (KF-CS). In the literature [6], [7], the CS theory has been used to UWB communication which exploits the sparse of the impulse response of the channel. The proposed method takes account of the prior knowledge that the impulse response is sparse in the time domain and provides estimation for the posterior density function of additive noise encounter when implementing the compressive measurement. Comparing with the conventional CS reconstruction algorithm, our approach takes the time-varying of the channel parameters into account and reduces the required sampling points and improves the accuracy of estimation algorithm.

The remainder of the paper is organized as follows. Section II gives a brief description of the UWB system and channel modeling. Section III depicts the details of the proposed KF-CS based IR-UWB channel estimation method. Our simulation results are given in Section IV. Section V gives the conclusion.

II. UWB SYSTEM AND CHANNEL MODELING

We consider the single user UWB system with direct sequence pulse amplitude modulation (DS-PAM-UWB). The transmitted signal can be expressed as

\[ s(t) = \sum_{k=0}^{+\infty} b_k g_p(t - kT_p) \]  

(1)
where \( b_k \) denotes the k-th information symbol taking value \( \pm 1 \) with equal probability, \( T_b \) is the symbol duration, \( g_T(t) \) is the transmitted symbol waveform and expressed as:

\[
g_T(t) = \sum_{j=0}^{N_f-1} c_j g(t - jT_f)
\]

where \( T_f \) denotes the frame period, \( N_f \) denotes the number of frames per symbol which satisfies \( T_b = N_f T_f \), \( c_j \) is the Pseudo-Noise code which period is \( N_f \), \( g(t) \) represents the elementary pulse. Consider the classical Saleh-Valenzuela channel model; the impulse response can be expressed as:

\[
h(t) = X \sum_{i=1}^{L} \sum_{k=1}^{K} \alpha_{kl} \delta(t - T_i - \tau_{ki})
\]

where \( \delta(\cdot) \) is the Dirac delta function, \( \alpha_{kl} \) is the tap weight of the k-th component in the l-th cluster, \( \tau_{ki} \) is the delay of the k-th multiple path component (MPC) relative to the l-th cluster arrival time \( T_i \). \( K \) is the number of the MPCs within a cluster. \( L \) is the number of the cluster. \( X \) is the channel path gain, which follow a log-normal distribution. \( \alpha_{kl} \) is positive or negative with equal probability and \( |\alpha_{kl}| \) follow a log-normal distribution. In this paper, we consider the IEEE 802.15.3a channel model; the channel impulse response can be expressed as:

\[
h(t) = \sum_{j=0}^{L-1} \gamma_j \delta(t - \tau_j)
\]

where \( \{\gamma_j\}_{j=0}^{L-1} \) and \( \{\tau_j\}_{j=0}^{L-1} \) denote channel fading coefficients and delays along different paths, respectively. \( L \) is the total number of channel taps.

Assuming the perfect synchronization at the receiver and the k-th observation with symbol-long can be represented as

\[
r_k(t) = b_k \sum_{j=0}^{L-1} \gamma_j g_T(t - \tau_j) + z_k(t), \quad t \in [0, T_b]
\]

where \( z_k(t) \) denotes the white Gaussian noise. To estimate \( h(t) \), the special training sequence is transmitted, i.e. \( b_k = 1 \), the sample period of receiver is \( T_r = T_p / 2 \). The number of samples within a period is \( N = [T_b / T_r] \). The discrete form of \( r_k(n) \) can be represented as:

\[
r_k(n) = g_T(n) \otimes h(n) + z_k(n), \quad n \in [0, N - 1]
\]

where \( g_T(n) \) and \( z_k(n) \) denote the discrete form of \( g_T(t) \) and \( z_k(t) \) respectively. \( h(n) \) denotes the discrete impulse response of \( h(t) \). \( \otimes \) is circle convolution. The matrix form of \( h(t) \) can be expressed as:

\[
r_k = Gh + z_k
\]

where

\[
G = \begin{bmatrix}
g_T(0) & g_T(N-1) & \cdots & g_T(1) \\
g_T(1) & g_T(0) & \cdots & g_T(2) \\
\vdots & \vdots & \ddots & \vdots \\
g_T(N-1) & g_T(N-2) & \cdots & g_T(0)
\end{bmatrix}
\]

\[
h = [h(0) \ h(1) \ \cdots \ h(N-1)]^T
\]

\[
z_k = [z_k(0) \ z_k(1) \ \cdots \ z_k(N-1)]^T
\]

\[
r_k = [r_k(0) \ r_k(1) \ \cdots \ r_k(N-1)]^T
\]

III. KALMAN FILTER COMPRESS SENSING AND CHANNEL MODELING

In UWB communications, an ultra-short duration pulse is used as the transmit pulses, typically on the order of nanoseconds, so the bandwidth of the signal occupancy is very wide. The system need very high sampling rate, but the hardware technology is very limited. At the same time, owing to the characteristic of the pulse waveform, the UWB channel is rich in multipath diversity, in the case of indoor environment, the multipath of UWB channel up to thousands of components, if all components were received and resolved, the computation will be very large and time exhaust, and also affect the accuracy of channel estimation. Although there are rich multipath components, in statistical, most of the multipath energy is concentrated in a small number of components, nearly 10% of the total number of channel multipath components own nearly 85% of the energy, so UWB channel is sparse.

Compressed sensing technology is a low-rate sampling and reconstruction process for sparse signal or compressible signal. First, the signal is projected to its sparse bases, down-sampling the signal by measurement matrix to obtain the observed data, the original signal can be reconstructed by linear optimization. The expressions of compressed sensing as follows:

\[
y_{N \times 1} = \Phi_{N \times N} x_{N \times 1} = \Phi_{N \times N} \Psi_{N \times N} \theta_{N \times 1}
\]

\[
x_{N \times 1} = \Psi_{N \times N}^T \theta_{N \times 1}
\]

where \( x_{N \times 1} \) indicated the compressible signal with length \( N \), the projection on orthogonal space(sparse bases) \( \Psi_{N \times N} \) is sparse vector \( \theta_{N \times 1} \), the non-zero coefficient is much smaller than the number of \( N \), \( S = ||\theta||_0 \), \( S << N \); \( y_{N \times 1} \) indicated the observation vector with length \( n \). According to the result in literature [5], under the condition that \( \Phi_{N \times N} \) and \( \Psi_{N \times N} \) satisfied the RIP (restricted isometry property, RIP), compressed sensing need only
$n = O(S \log(N/S))$ measurement points to recover compressible signal with sparse degree $S$ by linear optimization or greedy algorithm. In the literature [10], compressed sensing channel estimation method for UWB based random filter and circular convolution was proposed, the channel impulse response reconstructed by basis pursuit (BP) algorithm, although the sampling rate was effective reduced, but the complexity of the algorithm is not convenient to realize, while does not consider the case of time-varying channel state; [8-9] proposed sparse signal that varies slowly over time can be reconstructed by Kalman filtering compressed sensing reconstruction algorithm, and reduced the complexity of the algorithm, in this paper we use KF-CS algorithm for UWB channel estimation, and consider the time-varying channel state.

Figure 1 show the system structure, DS modulated signal directly into the UWB channel after the wave form. At the receiver, the measurement is obtained by down-sampling, and then to estimate the channel impulse response by KF-CS algorithm.

Conventional channel estimation algorithm usually assumes the channel impulse response is time invariant, but the actual environment is not the same, especially for channel estimation algorithm with the application of maximum likelihood, the computation of the process takes too long, the channel real information has changed. For the case of time-varying, we consider using Kalman filer to estimate the channel impulse response for different time. The actual channel states can be described by the first order autoregressive model, channel impulse response at different interval of symbols, can be expressed as:

$$h_k = h_{k-1} + w_{k-1}$$

(13)

where $w_{k-1} = [w_{k-1}(0) \cdots w_{k-1}(N-1)]^T$ indicated the process noise, assume that $w_{k-1}$ is independent and identically distributed zero mean Gaussian noise process and is independent with channel impulse response $h_{k-1}$, its covariance matrix is $Q$. Take (13) as the state equation and (7) as the measurement equation, the channel state $h_k$ can be estimated by Kalman filter. Equation (8) shows that the $N \times N$ gain matrix $G$ will make the computation of the algorithm very large. Literature [11] proposed frequency-domain pretreatment to simplify the iterative process, but the computation of $N$ points FFT is still large. If rewriting the observation equation by down-sampling, the problem would be converted to compressed sensing measurement and reconstruction process. The new measurement equation can be expressed as

$$r_k = D \downarrow (Gh_k + z_k)$$

(14)

where $D \downarrow$ indicated down-sampling, the sampling factor is $[N/n]$, note that the down-sampling is linear process, can be seen as mapping process from $h_k$ to $r_k$

$$r_k = Ch_k + v_k$$

(15)

where $C$ is $n \times N$ quasi-Toeplitz matrix, i.e. each row of $C$ is the previous row shift $[N/n]$ steps, and $C$ can be expressed as:

$$C = G_{\Omega}, \quad \Omega = \{N/n, 2N/n, 3N/n, \ldots nN/n\}$$

(16)

$G_{\Omega}$ denotes the sub-matrix obtained by extracting the columns of $G$ corresponding to the indices in $\Omega$. $v_k$ denotes the measurement noise with the covariance matrix $R$. The element of $C$ will be random due to the period sampling from the PN code. As literature [12-13], (15) is compressed sensing process. Take (13) as the state equation and (15) as the measurement equation, Kalman filter compress sensing iterative equation for UWB channel estimation can be expressed as:

$$\hat{h}_{k|k-1} = \hat{h}_{k-1}$$

$$P_{k|k-1} = P_{k-1} + Q$$

$$\hat{h}_k = \hat{h}_{k-1} + K_k (r_k - Ch_{k-1})$$

$$K_k = P_{k|k-1} C^H (CP_{k|k-1} C^H + R)^{-1}$$

$$P_k = (1 - K_k C) P_{k|k-1}$$

(17) (18) (19) (20) (21)

where subscript $k \mid k-1$ denotes the one-step prediction from k-1th symbol to k-th symbol. $h_k$ is the state vector and $\hat{h}_k$ is its prediction. $P_k$ stands for the $N \times N$ error covariance matrix of $h_k$:

$$P_k = E[(h_k - \hat{h}_k)(h_k - \hat{h}_k)^H]$$

(22)

$K_k$ represents the $N \times N$ Kalman gain matrix. Once the initial values $h_0$, $P_0$ and $Q$ are given, this iterative process continues until $k$ is equal to the length of training sequence.
Designed for the change slowly signal, KF-CS includes two parts: Kalman filter and sparse bases search. First, compressed sensing algorithm search the set of signal sparse bases $T_{k-1}$. The Kalman filter process will just work on the $T_{k-1}$, which mean a reduce order iterative. Next, FEN (Filtering error norm) will be calculated for detecting new bases of $T_{k-1}$. The Dantzig selector will search for the new set of the iterative if the FEN is larger than the threshold. For the long time remain near-zero coefficients, the bases will be removed. At last, the MMSE $\hat{h}_k$ and $T_k$ will be output. The KF-CS algorithm is summarized in Figure 2.

IV. SIMULATION RESULT

In this section we present our simulation results, which offer compelling evidence that the KF-CS is every suitable for the IR-UWB channel estimation. We used the IEEE 802.15.3a standard channel model to test the performance of algorithm. The channel impulse response satisfied first-order symbol regressive model. The second derivative of the Gaussian function was employ as elementary pulse. The impulse duration is $T_p = 1ns$. The duration of elementary pulse is $T_p = 1ns$, the sample period is 500 ps. The length of PN code is 12. The sampling factor is 4. The training sequence is $\hat{b}_k = 1$. The simulation is based on the Matlab R14.

Figure 3 compare the original channel impulse response and the estimation one, when the number of sample is just one fourth of the channel length. From the Figure 3, it is obviously that UWB multiple path components can be estimated precise under the low sampling rate. Figure 4 show the normalize MSE under different SNR for Kalman Filter and KF-CS. After finding out the sparse support set of the channel impulse, the MMSE solution was given by the KF-CS algorithm. This method reduced the computational complexity and avoided the compressed sensing recovery progress. The KF-CS algorithm outperformed the classical KF just because the estimation is given by the under-sample data. The KF algorithm is hardly estimate the signal from the measurement date without taking account the sparsity of the signal.

Figure 5 describe the normalize MSE under different channel change. The KF-CS channel estimation algorithm is proposed for time-varying channel. $S$ is defined the maxima difference of the channel sparse vector at adjacent time. Figure 5 show that the MSE performance will deteriorate as the change of the sparse signal become greater and fast. At the same time, the Dantzig Selector will be called for more times and the computation of the algorithm will increase.
V. CONCLUSIONS

In this paper, under the scene of channel state slowly changing, the Kalman filter compressed sensing algorithm was employed for channel estimating. By under-sampling the DS-PAM-UWB signal, we can not only reduce the number of sampling point but also establish the compressed sensing random measurement. The simulation results show that the proposed algorithm outperforms the traditional Kalman Filter under the condition that the channel changes slowly and the measurement point is less than the signal length. Once the channel change became severe, Dantzig Selector algorithm is called frequent in order to ensure the error performance, the computation increase correspondingly.

REFERENCES


